

Class Agenda

- Orbital Mechanics
- Keplerian Orbits
- Satellite Equations of Motion
 - Circular
 - Elliptical
 - Parabolic
 - Hyperbolic
- Hohmann Transfer
- Interplanetary Trajectories
 - Departure
 - Arrival
- Earth-Moon System

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Astrodynamics

- Astrodynamics – study of a satellite's trajectory/orbit
- Orbit – path through space
- Ephemeris – table listing its position as a function of time

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Orbital Mechanics

- 1800 B. C.
 - Inertial position of vernal equinox (Stonehenge)
- 350 B.C.
 - Aristotle – wandering motion of planets – universe composed of 55 concentric spheres centered in Earth
 - Each planet located in a sphere
- Aristarchus
 - Proposed sun and stars fixed – planets rotated around them – not accepted
- 150 A.D.
 - Ptolemy – elaborate Earth-centered theory
 - Tables used for 1400 years

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Orbital Mechanics

- 1543
 - Copernicus – sun-centered rotation
- 1610
 - Galileo – observations reinforced Copernicus
 - Observed Jupiter's moons orbiting Jupiter, not Earth
 - Observed moonlike phases of sunlight on Venus – not explained by Ptolemy
 - Forced to recant by Catholic Church
- Tyco Brahe
 - First accurate measurements of planet positions as a function of time
 - Allowed Kepler to describe mathematically heliocentric motion of planets

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Keplerian Orbits

- Kepler – described elliptical planetary orbits about Sun
- Newton – mathematical solution for system based on inverse-square gravitational force
- Kepler published his first two laws of planetary motion in 1609, third law in 1619

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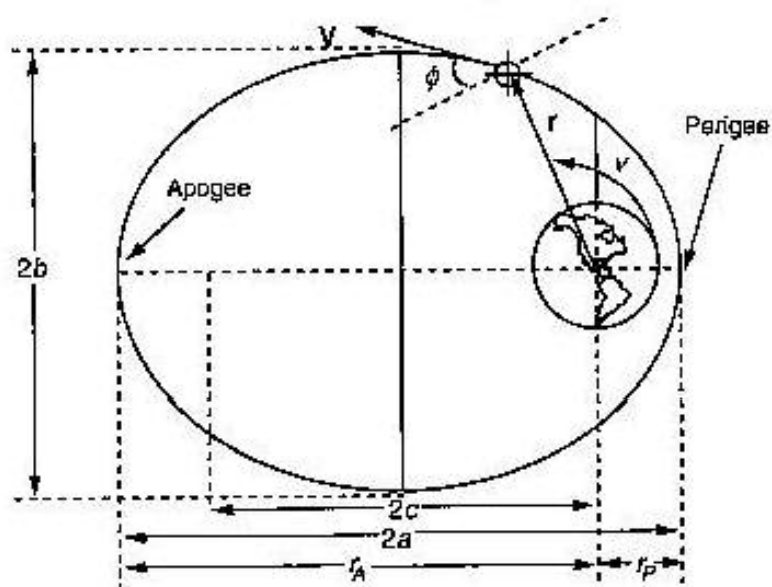
Keplerian Orbits

- 1st Law – orbit of each planet is an ellipse, with Sun at one focus
- 2nd Law – line joining planet and Sun sweeps out equal areas in equal times
- 3rd Law – square of period of a planet is proportional to cube of mean distance from Sun

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Satellite Equations of Motion



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Satellite Equations of Motion

- Newton's law of universal gravitation

$$F_g = \frac{MmG}{r^2}$$

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Satellite Equations of Motion

- Spacecraft motion governed by infinite network of attractions
- Dominated by one central body at a time
- Two-body assumptions
 - Motion of spacecraft is governed by attraction to a single central body
 - Mass of spacecraft is negligible compared to central body
 - Bodies are spherically symmetric with masses concentrated at centers
 - No forces act on bodies except for gravitational forces and centrifugal forces acting along center lines

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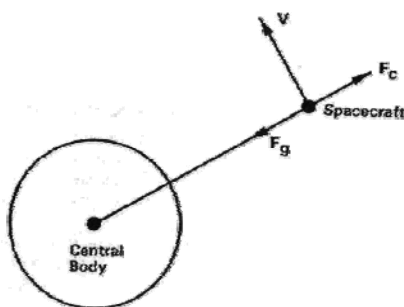
Satellite Equations of Motion

- If assumptions hold, conic sections are only possible paths for orbiting bodies, central body must be focus of the conic
- Assumptions nearly true
 - Oblateness of Earth leads to small errors

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Circular Orbits



- Centrifugal force on spacecraft

$$F_c = \frac{mV^2}{r}$$

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Circular Orbits

- For circular, steady-state motion

$$-F_g = F_c$$

$$\frac{mV^2}{r} = \frac{MmG}{r^2}$$

- Solving for v

$$V = \sqrt{\frac{MG}{r}}$$

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Circular Orbits

- Gravitational parameter, μ

$$- \mu = MG$$

- Therefore

$$V = \sqrt{\frac{\mu}{r}}$$

- μ is a property of central body – known for each of the planets

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Astrodynamic Parameters

Body	μ , km ³ /s ²	R_0 , km	A_z , deg/s	J_2	Mean solar distance, km $\times 10^6$
Mercury	22,032.1	2439.7	0.0000711		57.9
Venus	324,858.8	6051.8	-0.0000171	0.000027	108.2
Earth	398,600.4	6378.14	0.0041781	0.00108263	149.6
Mars	42,828.3	3397	0.0040613	0.001964	228.0
Jupiter	126,711,995.4	71,492	0.0100756	0.01475	778.4
Saturn	37,939,519.7	60,268	0.0093843	0.01645	1433
Uranus	5,780,158.5	25,559	-0.0058005	0.012	2883
Neptune	6,871,307.8	24,764	0.0062073	0.004	4517
Pluto	1020.9	1195	-0.0006524		5820
Moon	4902.8	1737.4	0.0001525	0.0002027	
Sun	132,712,439,935.5	696,000	0.0001642		

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Circular Orbits

- Circular orbit period

$$P = \frac{\text{circumference}}{\text{velocity}}$$

$$P = 2\pi \sqrt{\frac{r^3}{\mu}}$$

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General Solution

- Circular motion – special case of two-body motion
- General solution conclusions
 - Kepler's laws of planetary motion are confirmed
 - Sum of potential energy and kinetic energy of orbiting body, per unit mass, is constant at all points in the orbit

$$\mathcal{E} = \frac{V^2}{2} - \frac{\mu}{r}$$

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General Solution

- Can be reduced to

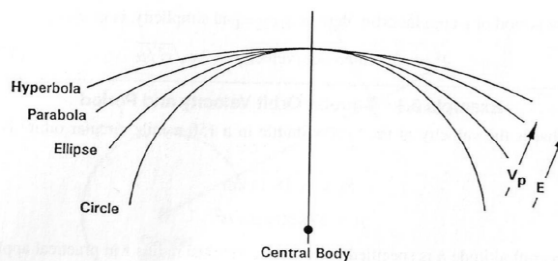
$$\mathcal{E} = -\frac{\mu}{2a}$$

- Total energy of orbit depends on the semimajor axis (a) only
- Circular orbit, $a=r$, $\mathcal{E} -$
- Elliptical orbit, $a +$, $\mathcal{E} -$
- Parabolic orbit, $a=\infty$, $\mathcal{E}=0$
- Hyperbolic orbit, $a -$, $\mathcal{E} +$

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UAH

General Solution

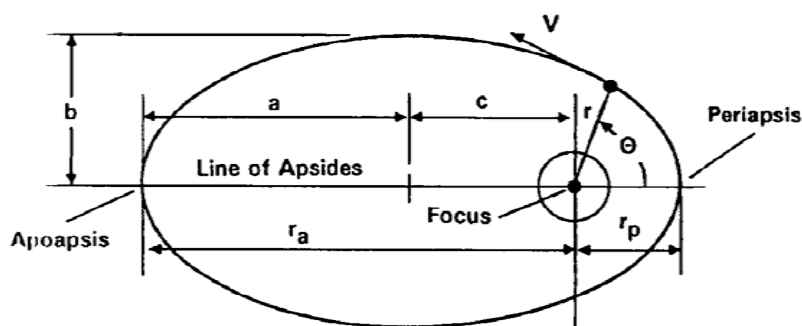


- Additional energy must be added to a spacecraft to change an orbit from circular to elliptical
- Energy must be removed to change from elliptical to circular

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UAH

General Solution



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General Solution

- Useful form

$$a = -\frac{\mu}{2\varepsilon}$$

- Total angular momentum of orbiting body
 - Constant
 - $H = r \times V$
 - $H = rV\cos\gamma$

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General Solution

- Eccentricity, e , defines shape of conic orbit

$$e = \frac{c}{a}$$

- $e = 0$ for circular orbit
- $e < 1$ for elliptical orbit
- $e = 1$ for parabolic orbit
- $e > 1$ for hyperbolic orbit

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General Solution

- Specific energy and eccentricity are related

$$e = \sqrt{1 - \frac{H^2}{\mu a}}$$

- General relation for velocity of orbiting body

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

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General Solution

- Circle, $a=r$

$$V = \sqrt{\frac{\mu}{r}}$$

- Ellipse, $a>0$

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

- Parabola, $a=\infty$

$$V = \sqrt{\frac{2\mu}{r}}$$

- Hyperbola, $a<0$

$$V = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}$$

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General Solution

- Equations can define orbit and discover type given only r , V , and θ
 1. Given r , V , calculate specific energy
 2. With specific energy, semimajor axis obtained
 3. Given r , V , and γ , magnitude of specific momentum obtained
 4. With specific momentum and semimajor axis, eccentricity obtained
 5. From characteristics of eccentricities, orbit type can be determined

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Elliptical Orbits

- Most common orbit
- All planets and most spacecraft move in
- Terms
 - a = semimajor axis
 - e = eccentricity
 - r_a = apoapsis radius
 - r_p = periapsis radius
- Periapsis – point of closest approach to central body (minimum radius)
- Apoapsis – point of maximum radius
- Line of apsides – apoapsis, periapsis, and center of mass line

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Elliptical Orbits

- Long axis – sum of apoapsis radius and periapsis radius

$$a = \frac{r_a + r_p}{2}$$

- Semimajor axis
 - Defines size of orbit
 - Indicates energy of orbit
 - Called the mean distance

$$c = \frac{r_a - r_p}{2}$$

- Distance between elliptical foci is $2c$

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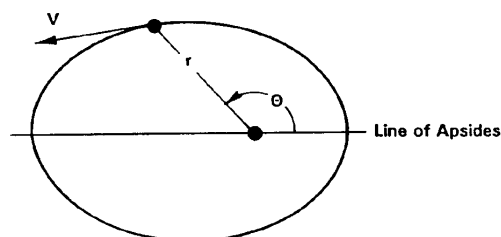
Elliptical Orbits

- If $e = \frac{c}{a}$

- Then $e = \frac{r_a - r_p}{r_a + r_p}$

- Semiminor axis, b , of ellipse is related
 $a^2 = b^2 + c^2$

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UAH***Elliptical Orbits***

- Given orbit defined by e and a

$$r = \frac{a(1 - e^2)}{(1 + e \cos \theta)} \qquad r = \frac{r_p(1 - e)}{(1 + e \cos \theta)}$$

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UAH***Elliptical Orbits***

- Given a defined orbit, true anomaly can be calculated

$$\cos \theta = \frac{r_p(1 + e)}{re} - \frac{1}{e} \qquad \cos \theta = \frac{a(1 + e^2)}{re} - \frac{1}{e}$$

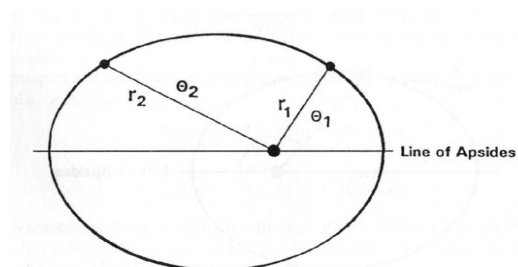
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UAH***Elliptical Orbits***

- Elliptical orbit through two given points

$$r_1 = \frac{r_p(1+e)}{1+e\cos\theta_1}$$

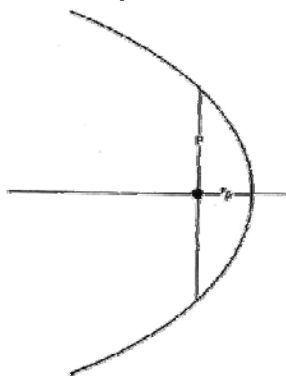
$$r_2 = \frac{r_p(1+e)}{1+e\cos\theta_2}$$



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UAH***Parabolic Orbit***

- Achieved by object falling from an infinite distance toward a central body
- Comets approach parabolic orbits



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Parabolic Orbit

- Parabola – considered an ellipse with an infinite semimajor axis
- Arms become parallel as r approaches infinity and when $e=1$ and $a = \infty$
- Velocity $V = \sqrt{\frac{2\mu}{r}}$
- Least energetic orbits
- Minimum velocity needed for a spacecraft to escape central body – escape velocity
 - Greater spacecraft altitude – lower escape velocity

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Hyperbolic Orbits

- Used for Earth departure on planetary flights
- Use for planetary arrival and targeting
- Use for energetic gravity-assist maneuvers that change direction and magnitude of spacecraft velocity without propulsion
- At any radius, a spacecraft on a hyperbolic orbit has greater velocity than it would on parabolic orbit – all hyperbolas are escape trajectories

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Hyperbolic Orbits

- Hyperbolic trajectory velocity > parabolic trajectory velocity
- Parabolic velocity goes to zero for infinite radius
- Hyperbolic – velocity is finite $V_{\infty} = \sqrt{\frac{\mu}{a}}$
- V_{∞} - velocity in excess of escape velocity – hyperbolic excess velocity (V_{HE})
- V_{HE} is velocity that must be added to Earth's velocity to achieve departure on a planetary mission

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Hyperbolic Orbits

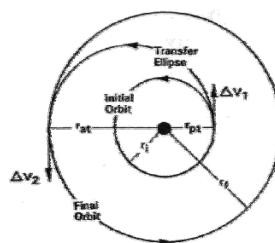
- Traditional to express as C3
 - $C3 = V_{HE}^2$
 - C3 used to describe hyperbolic departure from Earth, not used to describe arrival at planet

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Hohmann Transfer

- Transfer between two nonintersecting orbits
- Employs an elliptical transfer orbit tangent to initial and final orbit at apsides
- Design
 - Set periapsis radius of transfer ellipse equal to radius of initial orbit
 - Set apoapsis radius equal to radius of final orbit



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Hohmann Transfer

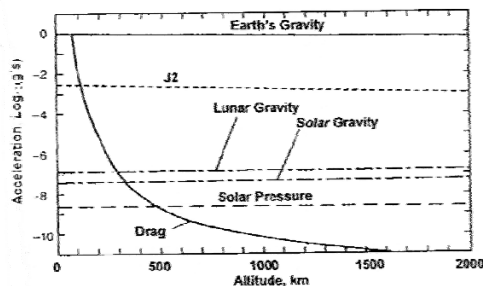
- Two velocity increments required
 - Changes initial velocity of spacecraft to velocity needed on transfer ellipse
 - $\Delta V_1 = V_{pt} - V_i$
 - Changes from velocity needed on transfer ellipse to velocity need on final orbit
 - $\Delta V_2 = V_{at} - V_f$
- Transfer can be between circular or elliptical orbits
- Transfer can be from high to low orbit
- Efficiency – two velocity changes are made at points of tangency between trajectories

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Orbit Perturbations

- Assumed mass of central body spherically symmetrical and concentrated at geometric center
- Assumed gravitational attraction is only force acting on spacecraft
- Perturbations
 - Oblateness of Earth
 - Drag
 - Attraction of sun
 - Attraction of moon
 - Solar radiation pressure



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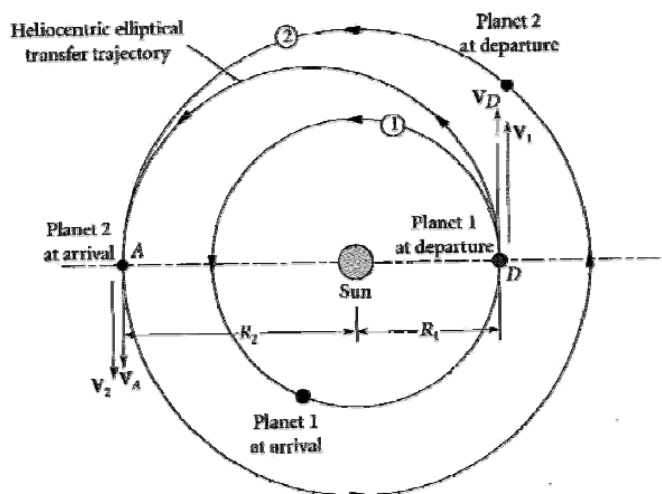
Earth's Oblateness

- Earth not spherically symmetric
- Equatorial radius – 6378.14 km
- Polar radius – 6356.77 km
- Caused by axial rotation rate of Earth
- Two major perturbations
 - Regression of nodes – orbit plane to precess gyroscopically (orbital rotation)
 - Rotation of apsides – rotation of periapsis

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Interplanetary Hohmann Transfers



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Interplanetary Hohmann Transfers

- Departure ΔV

$$\Delta V_D = V_D - V_1 = \sqrt{\frac{\mu_{sun}}{R_1}} \left(\sqrt{\frac{2R_2}{R_1 + R_2}} - 1 \right)$$

- Arrival ΔV

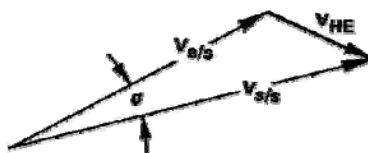
$$\Delta V_A = V_2 - V_A = \sqrt{\frac{\mu_{sun}}{R_2}} \left(1 - \sqrt{\frac{2R_1}{R_1 + R_2}} \right)$$

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Hyperbolic Excess Velocity and C3

- Hyperbolic excess velocity (V_{HE}) - Vector difference between velocity of Earth with respect to sun and velocity required on transfer ellipse
- Hyperbolic excess velocity (V_{∞}) on departure hyperbola
 - Excess amount above the escape velocity



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Hyperbolic Excess Velocity and C3

- V_{HE} is negative for a Venus mission (or any mission to an inner planet) – indicates Earth's orbital velocity must be reduced to enter the transfer ellipse
- V_{HE} – measure of energy required from launch vehicle system
- Traditional to use $C3 = V_{HE}^2$
 - Major performance parameter agreed on between launch vehicle and planetary spacecraft
- C3 – comes from mission design; represents minimum energy requirement needed to accomplish mission
- C3 – maximum energy launch vehicle can deliver carrying a spacecraft of a given weight

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V_∞ at the Planet

- When spacecraft arrives at target planet, a velocity condition analogous to departure occurs
- Hyperbolic excess velocity on arrival at planet is called V_∞ or V_{HP}
- V_∞ - vector difference between arrival velocity on transfer ellipse and orbital velocity of planet
- V_∞ - positive – indicating velocity must be reduced for capture

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Establishing Planetary Orbit

- Frequently desired to place spacecraft in orbit about target planet
- Establishing a planetary orbit requires simple orbit change
 - Velocity at periapsis of approach hyperbola

$$V_p = \sqrt{V_\infty^2 + \frac{2\mu}{r_p}}$$

- Velocity at periapsis of desired orbit

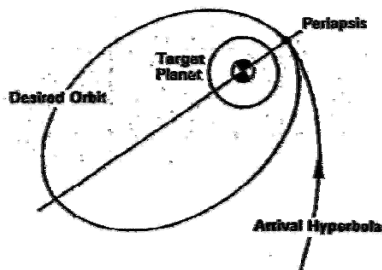
$$V'_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}}$$

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Establishing Planetary Orbit

- To put spacecraft into planetary orbit, velocity at periapsis must be reduced from V_p to V'_p
- Substantial spacecraft energy and weight are usually required
- Capture velocity of spacecraft must be reduced to a value below $\sqrt{2\mu/r}$

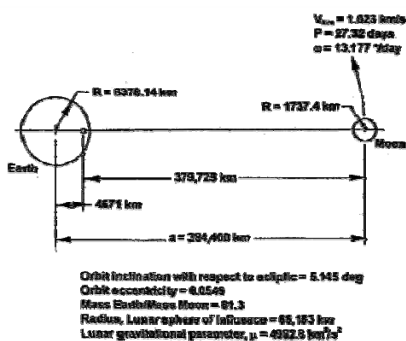


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Motion of Earth-Moon System

- Earth-moon system is unique
- Two bodies are so close to the same mass that, had the moon been slightly larger, they would be the only known binary planet system
- Common misconception – moon revolves around Earth
- Earth and moon revolve around a common center of mass – 4671 km from center of Earth and 379,729 km from center of moon
- One sidereal rotation about center takes 27.32 days



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**Example 1**

A satellite is placed in Earth orbit at an altitude of 700 km. The eccentricity of the orbit is 0. Find the following:

- a) Orbit type
- b) Velocity at the perigee
- c) Velocity at the apogee
- d) Orbit period
- e) If the satellite were placed incorrectly in a 600 km orbit, what would the satellite's velocity be?

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**Example 2**

A spacecraft has entered a circular orbit at Mars at an altitude of 35,000 km. Design a Hohman transfer to lower the orbit to an altitude of 1000 km. Provide all velocities required (ΔV_1 , ΔV_2) and the transfer time.

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