Class Agenda

- Orbital Mechanics
- Keplerian Orbits
- Satellite Equations of Motion
  - Circular
  - Elliptical
  - Parabolic
  - Hyperbolic
- Hohmann Transfer
- Interplanetary Trajectories
  - Departure
  - Arrival
- Earth-Moon System

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Orbital Mechanics

- 1800 B. C.
  - Inertial position of vernal equinox (Stonehenge)
- 350 B.C.
  - Aristotle – wandering motion of planets – universe composed of 55 concentric spheres centered in Earth
  - Each planet located in a sphere
- Aristarchus
  - Proposed sun and stars fixed – planets rotated around them – not accepted
- 150 A.D.
  - Ptolemy – elaborate Earth-centered theory
  - Tables used for 1400 years

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Orbital Mechanics

- 1543
  - Copernicus – sun-centered rotation
- 1610
  - Galileo – observations reinforced Copernicus
  - Observed Jupiter's moons orbiting Jupiter, not Earth
  - Observed moonlike phases of sunlight on Venus – not explained by Ptolemy
  - Forced to recant by Catholic Church
- Tyco Brahe
  - First accurate measurements of planet positions as a function of time
  - Allowed Kepler to describe mathematically heliocentric motion of planets

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Keplerian Orbits

- Kepler – described elliptical planetary orbits about Sun
- Newton – mathematical solution for system based on inverse-square gravitational force
- Kepler published his first two laws of planetary motion in 1609, third law in 1619

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Keplerian Orbits

- 1st Law – orbit of each planet is an ellipse, with Sun at one focus
- 2nd Law – line joining planet and Sun sweeps out equal areas in equal times
- 3rd Law – square of period of a planet is proportional to cube of mean distance from Sun

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Satellite Equations of Motion

- Newton's law of universal gravitation

$$F_g = \frac{MmG}{r^2}$$

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Satellite Equations of Motion

- Spacecraft motion governed by infinite network of attractions
- Dominated by one central body at a time
- Two-body assumptions
  - Motion of spacecraft is governed by attraction to a single central body
  - Mass of spacecraft is negligible compared to central body
  - Bodies are spherically symmetric with masses concentrated at centers
  - No forces act on bodies except for gravitational forces and centrifugal forces acting along center lines

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Satellite Equations of Motion

- If assumptions hold, conic sections are only possible paths for orbiting bodies, central body must be focus of the conic
- Assumptions nearly true
  - Oblateness of Earth leads to small errors

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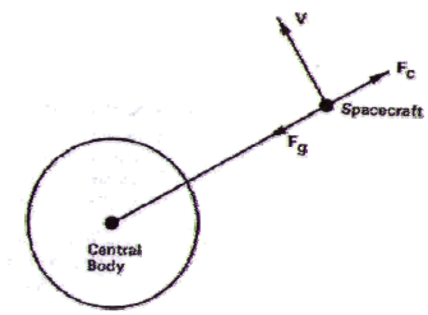
Circular Orbits



- Centrifugal force on spacecraft

$$F_c = \frac{mV^2}{r}$$

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Circular Orbits

- For circular, steady-state motion
  - $F_g = F_c$

$$\frac{mV^2}{r} = \frac{MmG}{r^2}$$

- Solving for v

$$V = \sqrt{\frac{MG}{r}}$$

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Circular Orbits

- Gravitational parameter, μ
  - $\mu = MG$
- Therefore

$$V = \sqrt{\frac{\mu}{r}}$$

- μ is a property of central body – known for each of the planets

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Orbital Mechanics Data

Appendix B.5, pg. 592

Body	μ, km^3/s^2	R_0, km	A_r, deg/s	J_2	Mean solar distance, km $\times 10^6$
Mercury	22,032.1	2439.7	0.0000711		57.9
Venus	324,858.8	6051.8	−0.0000171	0.000027	108.2
Earth	**398,600.4**	**6378.14**	**0.0041781**	**0.00108263**	**149.6**
Mars	42,828.3	3397	0.0040613	0.001964	228.0
Jupiter	126,711,995.4	71,492	0.0100756	0.01475	778.4
Saturn	37,939,519.7	60,268	0.0093843	0.01645	1433
Uranus	5,780,158.5	25,559	−0.0058005	0.012	2883
Neptune	6,871,307.8	24,764	0.0062073	0.004	4517
Pluto	1020.9	1195	−0.0006524		5820
Moon	4902.8	1737.4	0.0001525	0.0002027	
Sun	132,712,439,935.5	696,000	0.0001642		

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Circular Orbits

- Circular orbit period

$$P = \frac{circumference}{velocity}$$

$$P = 2\pi\sqrt{\frac{r^3}{\mu}}$$

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General Solution

- Circular motion – special case of two-body motion
- General solution conclusions
  - Kepler's laws of planetary motion are confirmed
  - Sum of potential energy and kinetic energy of orbiting body, per unit mass, is constant at all points in the orbit

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{r}$$

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General Solution

- Can be reduced to

$$\varepsilon = -\frac{\mu}{2a}$$

- Total energy of orbit depends on the semimajor axis (a) only
- Circular orbit, a=r, ε –
- Elliptical orbit, a +, ε –
- Parabolic orbit, a=∞, ε=0
- Hyperbolic orbit, a -, ε +

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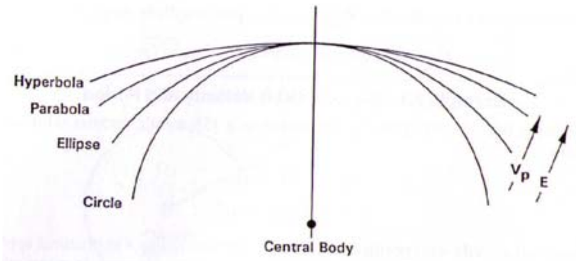
General Solution



- Additional energy must be added to a spacecraft to change an orbit from circular to elliptical
- Energy must be removed to change from elliptical to circular

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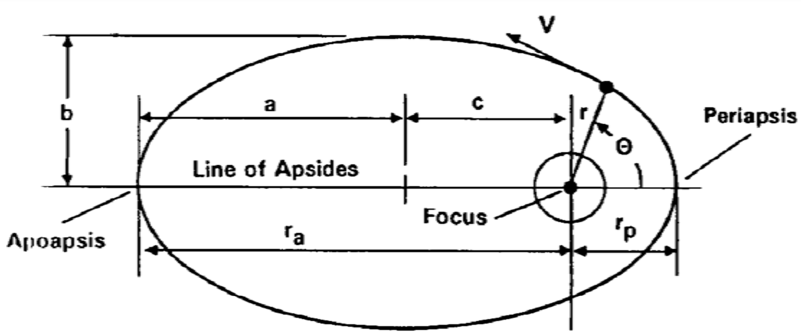
General Solution



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General Solution

- Useful form

$$a = -\frac{\mu}{2\varepsilon}$$

- Total angular momentum of orbiting body
  - Constant
  - H = r X V
  - H = rVcosγ

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General Solution

- Eccentricity, e, defines shape of conic orbit

$$e = \frac{c}{a}$$

- e = 0 for circular orbit
- e <1 for elliptical orbit
- e = 1 for parabolic orbit
- e > 1 for hyperbolic orbit

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General Solution

- Specific energy and eccentricity are related

$$e = \sqrt{1 - \frac{H^2}{\mu a}}$$

- General relation for velocity of orbiting body

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

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General Solution

- Circle, a=r

$$V = \sqrt{\frac{\mu}{r}}$$

- Ellipse, a>0

$$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

- Parabola, a=∞

$$V = \sqrt{\frac{2\mu}{r}}$$

- Hyperbola, a<0

$$V = \sqrt{\frac{2\mu}{r} + \frac{\mu}{a}}$$

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General Solution

- Equations can define orbit and discover type given only r, V, and θ
  1. Given r, V, calculate specific energy
  2. With specific energy, semimajor axis obtained
  3. Given r, V, and γ, magnitude of specific momentum obtained
  4. With specific momentum and semimajor axis, eccentricity obtained
  5. From characteristics of eccentricities, orbit type can be determined

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Elliptical Orbits

- Most common orbit
- All planets and most spacecraft move in
- Terms
  - a = semimajor axis
  - e = eccentricity
  - r_a = apoapsis radius
  - r_p = periapsis radius
- Periapsis – point of closest approach to central body (minimum radius)
- Apoapsis – point of maximum radius
- Line of apsides – apoapsis, periapsis, and center of mass line

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Elliptical Orbits

- Long axis – sum of apoapsis radius and periapsis radius

$$a = \frac{r_a + r_p}{2}$$

- Semimajor axis
  - Defines size of orbit
  - Indicates energy of orbit
  - Called the mean distance

$$c = \frac{r_a - r_p}{2}$$

- Distance between elliptical foci is 2c

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Elliptical Orbits

- If $e = \frac{c}{a}$
- Then $e = \frac{r_a - r_p}{r_a + r_p}$
- Semiminor axis, b, of ellipse is related

  $a^2 = b^2 + c^2$

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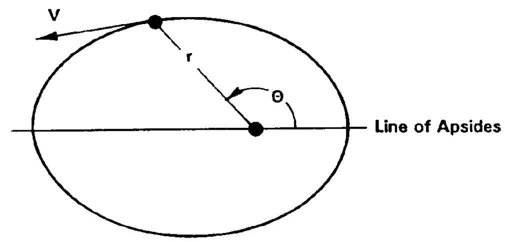
Elliptical Orbits



- Given orbit defined by e and a

$$r = \frac{a(1-e^2)}{(1+e\cos\theta)} \qquad r = \frac{r_p(1+e)}{(1+e\cos\theta)}$$

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Elliptical Orbits

- Given a defined orbit, true anomaly can be calculated

$$\cos\theta = \frac{r_p(1+e)}{re} - \frac{1}{e} \qquad \cos\theta = \frac{a(1+e^2)}{re} - \frac{1}{e}$$

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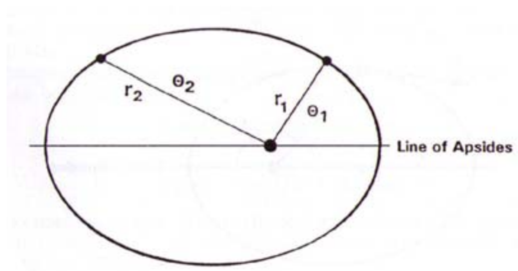
Elliptical Orbits

- Elliptical orbit through two given points

$$r_1 = \frac{r_p(1+e)}{1+e\cos\theta_1} \qquad r_2 = \frac{r_p(1+e)}{1+e\cos\theta_2}$$



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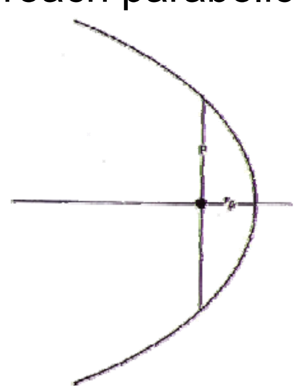
Parabolic Orbit

- Achieved by object falling from an infinite distance toward a central body
- Comets approach parabolic orbits



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Parabolic Orbit

- Parabola – considered an ellipse with an infinite semimajor axis
- Arms become parallel as r approaches infinity and when e=1 and a = ∞
- Velocity $V = \sqrt{\frac{2\mu}{r}}$
- Least energetic orbits
- Minimum velocity needed for a spacecraft to escape central body – escape velocity
  - Greater spacecraft altitude – lower escape velocity

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Hyperbolic Orbits

- Used for Earth departure on planetary flights
- Use for planetary arrival and targeting
- Use for energetic gravity-assist maneuvers that change direction and magnitude of spacecraft velocity without propulsion
- At any radius, a spacecraft on a hyperbolic orbit has greater velocity than it would on parabolic orbit – all hyperbolas are escape trajectories

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Hyperbolic Orbits

- Hyperbolic trajectory velocity > parabolic trajectory velocity
- Parabolic velocity goes to zero for infinite radius
- Hyperbolic – velocity is finite

$$V_\infty = \sqrt{\frac{\mu}{a}}$$

- V_∞ - velocity in excess of escape velocity – hyperbolic excess velocity (V_{HE})
- V_{HE} is velocity that must be added to Earth's velocity to achieve departure on a planetary mission

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Hyperbolic Orbits

- Traditional to express as C3
  - $C3 = V_{HE}^2$
  - C3 used to describe hyperbolic departure from Earth, not used to describe arrival at planet

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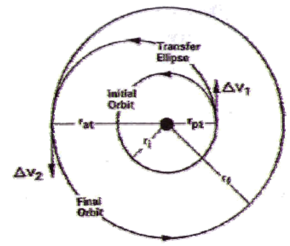
Hohmann Transfer

- Transfer between two nonintersecting orbits
- Employs an elliptical transfer orbit tangent to initial and final orbit at apsides
- Design
  - Set periapsis radius of transfer ellipse equal to radius of initial orbit
  - Set apoapsis radius equal to radius of final orbit



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Hohmann Transfer

- Two velocity increments required
  - Changes initial velocity of spacecraft to velocity needed on transfer ellipse
    - $\Delta V_1 = V_{pt} - V_i$
  - Changes from velocity needed on transfer ellipse to velocity need on final orbit
    - $\Delta V_2 = V_{at} – V_f$
- Transfer can be between circular or elliptical orbits
- Transfer can be from high to low orbit
- Efficiency – two velocity changes are made at points of tangency between trajectories

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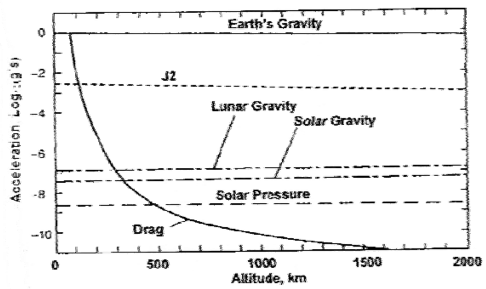
Orbit Perturbations

- Assumed mass of central body spherically symmetrical and concentrated at geometric center
- Assumed gravitational attraction is only force acting on spacecraft
- Perturbations
  - Oblateness of Earth
  - Drag
  - Attraction of sun
  - Attraction of moon
  - Solar radiation pressure



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Earth's Oblateness

- Earth not spherically symmetric
- Equatorial radius – 6378.14 km
- Polar radius – 6356.77 km
- Caused by axial rotation rate of Earth
- Two major perturbations
  - Regression of nodes – orbit plane to precess gyroscopically (orbital rotation)
  - Rotation of apsides – rotation of periapsis

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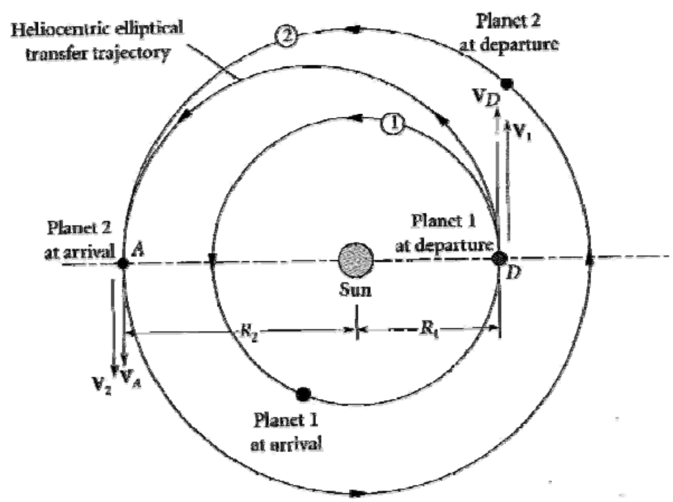
UAH Interplanetary Hohmann Transfers



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UAH Interplanetary Hohmann Transfers

- Departure ΔV

$$\Delta V_D = V_D - V_1 = \sqrt{\frac{\mu_{sun}}{R_1}}\left(\sqrt{\frac{2R_2}{R_1+R_2}}-1\right)$$

- Arrival ΔV

$$\Delta V_A = V_2 - V_A = \sqrt{\frac{\mu_{sun}}{R_2}}\left(1-\sqrt{\frac{2R_1}{R_1+R_2}}\right)$$

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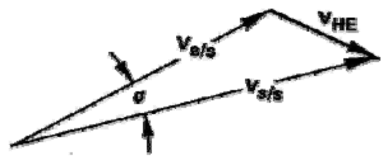
Hyperbolic Excess Velocity and C3

- Hyperbolic excess velocity (V_{HE}) - Vector difference between velocity of Earth with respect to sun and velocity required on transfer ellipse
- Hyperbolic excess velocity (V_{∞}) on departure hyperbola
  - Excess amount above the escape velocity



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Hyperbolic Excess Velocity and C3

- V_{HE} is negative for a Venus mission (or any mission to an inner planet) – indicates Earth's orbital velocity must be reduced to enter the transfer ellipse
- V_{HE} – measure of energy required from launch vehicle system
- Traditional to use C3 = V_{HE}^2
  - Major performance parameter agreed on between launch vehicle and planetary spacecraft
- C3 – comes from mission design; represents minimum energy requirement needed to accomplish mission
- C3 – maximum energy launch vehicle can deliver carrying a spacecraft of a given weight

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V_∞ at the Planet

- When spacecraft arrives at target planet, a velocity condition analogous to departure occurs
- Hyperbolic excess velocity on arrival at planet is called V_∞ or V_{HP}
- V_∞ - vector difference between arrival velocity on transfer ellipse and orbital velocity of planet
- V_∞ - positive – indicating velocity must be reduced for capture

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Establishing Planetary Orbit

- Frequently desired to place spacecraft in orbit about target planet
- Establishing a planetary orbit requires simple orbit change
  - Velocity at periapsis of approach hyperbola

$$V_p = \sqrt{V^2{}_\infty + \frac{2\mu}{r_p}}$$

  - Velocity at periapsis of desired orbit

$$V'_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}}$$

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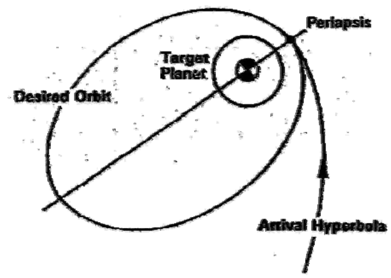
Establishing Planetary Orbit

- To put spacecraft into planetary orbit, velocity at periapsis must be reduced from V_p to V'_p
- Substantial spacecraft energy and weight are usually required
- Capture velocity of spacecraft must be reduced to a value below $\sqrt{2\mu/r}$



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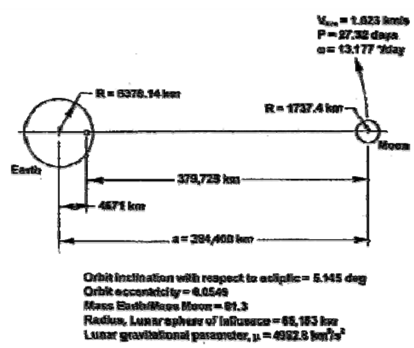
Motion of Earth-Moon System

- Earth-moon system is unique
- Two bodies are so close to the same mass that, had the moon been slightly larger, they would be the only known binary planet system
- Common misconception – moon revolves around Earth
- Earth and moon revolve around a common center of mass – 4671 km from center of Earth and 379,729 km from center of moon
- One sidereal rotation about center takes 27.32 days



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